rate. The faster propagation rates predicted by the model in Section 2.2 show good agreement with data in water. In nitrogen, the data are higher even though they exhibit significant scatter. For both liquids and gases, however, equation (8) provides a better estimate of the one-dimensional transport termination times than any other previous criteria.

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A note on the solution of the free convection boundary layer flow in a saturated porous medium

T. GOVINDARAJULU and G. MALARVIZHI

Department of Mathematics, Madras Institute of Technology, Madras 600 044, India

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INTRODUCTION

BUOYANCY driven flow past bodies immersed in a saturated porous medium has been the subject of the studies by Cheng [1–3]. The effect of uniform mass flux on the free convection boundary layer on a vertical wall in a saturated porous medium was studied by Merkin [4]. Cheng [5] presented a similarity solution for the case of wall temperature and suction velocity varying as powers of x, the longitudinal distance. In all these problems numerical solutions have been given for selected values of a parameter. In this note, an analytical series solution based on Von Mises transformation is given for the problem studied by Cheng [5]. It is found that even a few terms of the series are sufficient to yield the numerical results reported in ref. [5]. The treatment is similar to that of Merkin [6]. However, the equation and the boundary conditions are different.

THE EQUATION AND SERIES SOLUTION

The boundary layer equations of momentum and energy for the boundary layer flow past a vertical plate embedded in a saturated porous medium can be reduced to the form [5]

$$f''-\theta' = 0 \tag{1}$$

$$\theta''+\frac{1+\lambda}{2}f\theta'-\lambda f'\theta = 0 \tag{2}$$

where the plate temperature and suction or injection velocity are respectively given by

$$T_w - T_\infty = Ax^\lambda \tag{3}$$

and

$$v_w = ax^n \tag{4}$$

where

$$n = (\lambda-1)/2.$$

The boundary conditions are

$$\begin{aligned}\eta = 0:&\quad \theta = 1,\quad f = f_w\\ \eta\to\infty:&\quad \theta = 0,\quad f' = 0\end{aligned} \tag{5}$$

where f_w is the non-dimensional form of suction ($f_w > 0$) or injection ($f_w < 0$) velocity. Eliminating θ gives

$$f'''+\frac{1+\lambda}{2}ff''-\lambda(f')^2 = 0. \tag{6}$$

Following Merkin [6], we transform equation (6), expressing $p = p(\phi)$ where

$$p = f'(\eta),\quad \phi = c-f(\eta),\quad c = f(\infty). \tag{7a–c}$$

The modified equation is

$$\frac{d}{d\phi}\left(p\frac{dp}{d\phi}\right)+\frac{1+\lambda}{2}(\phi-c)\frac{dp}{d\phi}-p = 0. \tag{8}$$

The boundary conditions on p are

$$\begin{aligned}\phi = 0:&\quad p = 0\\ \phi = c-f_w:&\quad p = 1.\end{aligned} \tag{9}$$

We expand p in the series form

$$p = \sum_i a_i\phi^i. \tag{10}$$

NOMENCLATURE

A	constant defined in equation (3)
a	constant defined in equation (4)
a_i	coefficients in series (10)
c	constant defined in equation (7c)
f	dimensionless stream function
f_w	lateral mass flux parameter
n	constant defined in equation (4)
p	function defined in equation (7a)
T	temperature
v	suction velocity
x	longitudinal distance.

Greek symbols

η	dimensionless lateral coordinate
θ	dimensionless temperature
λ	constant defined in equation (3)
ϕ	independent variable defined in equation (7b).

Subscripts

∞	condition at infinity
w	condition at the plate.

Substituting this series in equation (8) and equating various coefficients of powers of ϕ to zero we get the recurrence formula for the coefficients

$$(s+1)^2c(\lambda+1)a_{s+1} = -2(s+1) \times \sum_{m=2}^{s} [ma_m a_{s-m+2}+(2\lambda-(1+\lambda)s)a_s], \quad s = 2, 3, 4, \ldots \qquad (11)$$

and

$$a_1 = \frac{c}{2}(1+\lambda), \quad a_2 = \frac{\lambda-1}{8}. \qquad (12)$$

The coefficients up to a_7 are explicitly given in the appendix. These coefficients decrease rapidly suggesting fast convergence of series (10). To find the value of c we use the boundary condition $p(c-f_w) = 1$. Thus

$$\sum_{s=1}^{\infty} a_s(c-f_w)^s = 1. \qquad (13)$$

For finite s, this is a simple algebraic equation for c. Given f_w, this can be solved by the interval halving method. c should be positive; this follows from the asymptotic form of the differential equation valid for $\eta \to \infty$

$$f''' + \frac{1+\lambda}{2}cf'' = 0. \qquad (14)$$

Then $\lambda \geqslant 0$ and $f''(\infty) = 0$ implies $c > 0$. Again using the equation

$$c-f_w = \int_0^{\infty} f'(\eta)\,\mathrm{d}\eta = \int_0^{\infty} \theta(\eta)\,\mathrm{d}\eta \qquad (15)$$

and assuming monotonic decay of temperature within the boundary layer, it follows that for suction, $c > f_w > 0$.

In most of the cases there is only one positive root, irrespective of the number of terms taken in equation (13). However, in a few cases there are at most two positive roots depending on the number of terms, with one root near the origin. In such cases, for $f_w > 0$ the root satisfying $c > f_w > 0$ and for $f_w < 0$ the common root valid for all the terms of the series, is considered.

Integration of equation (6) together with the relation

$$\int_0^{\infty} (f')^2\,\mathrm{d}\eta = \int_0^{c-f_w} p\,\mathrm{d}\phi \qquad (16)$$

gives

$$f''(0) = -\frac{1}{2}\left[(1+\lambda)f_w+(1+3\lambda)\sum_{s=1}^{\infty}\frac{a_s(c-f_w)^{s+1}}{s+1}\right]. \qquad (17)$$

The value of $f''(0)$ and c are given in Table 1. Cheng [5] showed that the range of λ for which the problem is physically

Table 1. Values of $-\theta'(0) = -f''(0)$ (N is the number of terms of the series used)

		0					1/3			
		Series		Numerical			Series		Numerical	
f_w	N	c	$-f''(0)$	c	$-f''(0)$	N	c	$-f''(0)$	c	$-f''(0)$
−4	20	1.23750	0.00265	1.23935	0.00305	16	0.61758	0.05131	0.62906	0.12499
−3	20	1.24490	0.01822	1.24491	0.01823	16	0.69234	0.16639	0.69234	0.16640
−2	8	1.27174	0.07211	1.27176	0.07213	7	0.79118	0.24381	0.79114	0.24372
−1	8	1.36640	0.20404	1.36635	0.20404	5	0.96768	0.39702	0.96760	0.39700
−0.8	7	1.40067	0.24289	1.40066	0.24291	5	1.01889	0.44154	1.01886	0.44153
−0.6	7	1.44204	0.28631	1.44204	0.28633	5	1.07720	0.49164	1.07719	0.49164
−0.4	6	1.49127	0.33429	1.49128	0.33431	5	1.14337	0.54759	1.14336	0.54759
−0.2	5	1.54910	0.38680	1.54912	0.38682	5	1.21802	0.60598	1.21802	0.60958
0	5	1.61611	0.44374	1.61612	0.44375	5	1.30163	0.67765	1.30163	0.67765
0.2	4	1.69272	0.50494	1.69265	0.50490	3	1.39445	0.75169	1.39448	0.75172
0.4	4	1.77888	0.57006	1.77884	0.57004	3	1.49659	0.83160	1.49660	0.83161
0.6	4	1.87465	0.63889	1.87463	0.63888	3	1.60779	0.91700	1.60780	0.91701
0.8	4	1.97974	0.71111	1.97972	0.71110	3	1.72769	1.00752	1.72769	1.00753
1	4	2.09367	0.78641	2.09366	0.78640	3	1.85573	1.10274	1.85574	1.10274
2	3	2.77412	1.19825	2.77409	1.19824	3	2.59480	1.63357	2.59480	1.63357
3	3	3.58179	1.64747	3.58179	1.64747	3	3.44274	2.22256	3.44274	2.22255
4	2	4.46028	2.11608	4.46021	2.11606	2	4.34844	2.84146	4.34842	2.84145

realistic is $0 \leqslant \lambda \leqslant 1$. Special cases are:

$\lambda = 0$: uniform wall temperature with $v_w \sim x^{-1/2}$

$\lambda = \frac{1}{3}$: uniform heat flux with $v_w \sim x^{-1/3}$

$\lambda = 1$: uniform wall velocity with $T_w \sim x$.

For the purpose of comparison, the problem was again solved numerically to five decimal places for selected values of λ and f_w, using the shooting method and the Runge–Kutta–Gill algorithm. These values are also shown in the table. For $\lambda = 1$, the coefficients $a_i = 0$ for $i \geqslant 2$ and the series gives the exact solution, which was not noticed by Cheng

$$f(\eta) = c - \frac{1}{c}e^{-c\eta} \tag{18}$$

where

$$c(c - f_w) = 1. \tag{19}$$

Keeping the first two terms of series (10) the approximate solution is found to be

$$f(\eta) = c - \frac{a_1(c - f_w)e^{-a_1\eta}}{a_1 + a_2(c - f_w)(1 - e^{-a_1\eta})} \tag{20}$$

where

$$c^2(5\lambda + 3) - 2cf_w(3\lambda + 1) + [f_w^2(\lambda - 1) - 8] = 0. \tag{21}$$

We find from the table that the series solution gives very good results with 5–8 terms. For positive and large values of f_w even 2 or 3 terms of the series are sufficient. However, for large negative values of f_w, considerably more terms are required. Moreover, in this case, $f''(0)$ is found to be sensitive to the values of c and the series solution is of limited use.

CONCLUSION

An approximate series solution is found for the free convection boundary layer flow past a vertical plate in a porous medium for $0 \leqslant \lambda \leqslant 1$ and for a wide range of the values of the wall suction and injection velocity f_w. For the case of suction relatively fewer terms are sufficient to give values which are in good agreement with the numerically computed results. The series solution also gives the exact solution for $\lambda = 1$.

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APPENDIX

$$a_3 = \frac{-(\lambda - 1)(3\lambda + 1)}{144c(\lambda + 1)}$$

$$a_4 = \frac{(\lambda - 1)(9\lambda^2 - 1)}{1152c^2(\lambda + 1)^2}$$

$$a_5 = \frac{-(603\lambda^4 - 822\lambda^3 + 112\lambda^2 + 118\lambda - 11)}{172\,800c^3(\lambda + 1)^3}$$

$$a_6 = \frac{(3501\lambda^5 - 6117\lambda^4 + 2186\lambda^3 + 734\lambda^2 - 295\lambda - 9)}{2\,073\,600c^4(\lambda + 1)^4}$$

$$a_7 = -(521\,937\lambda^6 - 1\,119\,366\lambda^5 + 639\,891\lambda^4 + 46\,556\lambda^3 - 97\,353\lambda^2 + 6762\lambda + 1573)/(609\,638\,400c^5(\lambda + 1)^5).$$